2014/35

Transferable and non transferable utility implementations of coalitional stability in integrated assessment models

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DISCUSSION PAPER

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Transferable and non transferable utility implementations of coalitional stability in integrated assessment models

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August 2014

Abstract

To study the stability of coalitions in the standard game theoretic model of international environmental agreements, two alternative concepts are used: potential internal stability and core stability. Both concepts make use of the possibility of reallocating payoffs within a coalition through transfers, formulated in terms of transferable utility among the players. For international applications where players are countries, such as done in the growing literature on integrated assessment models, non-transferable utility games would be economically better suited. In this note, we provide a framework for comparing the treatment of coalitions in five game theoretically minded integrated assessment models, from that point of view. Under way, we extend the definition of the two stability concepts to games without transferable utility, assuming instead the transferability of some physical good. We also show that potential internal stability and blocking power of coalitions can be tested by solving a simple optimization problem.

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This paper is one of the products of an informal group on coalition models comparison that met at PIK, Potsdam in February 2012 and at FEEM, Venice, in January 2013. Another paper from that group (Lessmann et al., 2014) is mentioned in the references section. Thanks are due to the organizers of the two meetings, as well as to the institutions that financed them. Tulkens is indebted to both FEEM and CORE for their support of the present research.

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1 Introduction

In the game theoretically inspired literature, that deals with the stability of international environmental agreements in view of which countries form coalitions, two alternative concepts are considered. Potential internal stability (PIS) on the one hand, introduced by Carraro et al. (2006), is a property that holds in any coalition if the agreement is such that a transfer scheme exists that guarantees all members at least their outside payoff. Core stability (CS) on the other hand, introduced in the field by Chander and Tulkens (1995, 1997), is a property that prevails in the grand coalition if no other coalition can provide each of its members a higher payoff when deviating from an agreement proposed for the grand coalition.

Both concepts, which are formulated analytically in terms of transferable utility (TU) games, make use of transfers among players. Using this instrument, Weikard (2009) defines the class of transfers necessary to induce PIS. Similarly, Chander and Tulkens (1995, 1997) give a formula of transfers that they show to be sufficient to induce CS in a wide class of international environmental games.

With Eyckmans and Tulkens (2003), the CS concept has been introduced in a numerical simulation model of the world economy dubbed CWS, with a view of computing the international transfers asserted to ensure the core stability of possible agreements. Since then, other authors have done the same with the PIS concept, introducing it in other simulation models, namely Nagashima et al. (2009) with STACO and Bosetti et al. (2013) with WITCH. The latter paper introduces extensions to non-transferable utility (NTU).

Obviously, an interesting comparison exercise focusing on the coalition stability issue was called for. Lessmann et al. (2014) analyze five integrated assessment models (IAMs) with respect to stability: MICA (Lessmann and Edenhofer, 2011), STACO (Finus et al., 2006), CWS (Eyckmans and Tulkens, 2003), WITCH (Bosetti et al., 2006), and RICE (Yang, 2008). To enlighten the results of this first comparison paper as well as future ones, the present note aims at making explicit some aspects of the underlying theory and of computational methodologies involved: while these are basically common to all five models, there are nevertheless important differences that deserve to be pointed out to make the comparability credible.

Our main concern is with the formulation of transfers between players in the alternative frameworks of TU vs NTU. Furthermore, as the economic models underlying the game differ considerably in terms of their scope, we aim here at a unifying presentation.

After reviewing in Section 2 the basic structure of IAMs, showing how strategic and coalitional games are associated with them and reminding one of the source of TU vs NTU distinction, we define in Section 3 the two stability concepts in their TU version and describe the role of transfers in the achievement of coalitional stability in either case. We extend the definitions in Section 4 to the NTU form of the game, substituting transfers of a standard economic commodity for transfers of utilities. We verify the extent to which results known for the game in the TU form do still hold in the NTU case. We finally propose in Section 5 methods for testing the stability of coalitions by means of a simple optimization problem. Section 6 discusses briefly the practicality of transfers.

2 The IAMs and their associated games

2.1 The worldwide economic-environmental models

This section introduces the generic IAM to which the descriptions of this work apply, following the well-known RICE-model (Nordhaus and Yang, 1996). The specific IAM may differ in model setup concerning decision variables and constraints.

Consider an international economy with environmental externalities, consisting of $n \in \mathbb{N}$ countries indexed by $i \in N \subset \mathbb{N}$ and whose activities (consumption, labor, investment, capital accumulation) extend over T periods, indexed $t \in \{1, \dots, \mathrm{T}\} = T$. Within that framework, the characteristic feature of integrated assessment models is to include the environmental dimension of the activities, namely emissions of pollutants and reception of ambient pollutants.

The general economic activities are supposed to be those of market economies, and the resulting emitted pollutants in all countries are assumed to accumulate over time as a single stock affecting all countries around the planet, hence a worldwide environmental and stock externality. This also reflects the models' intended application to climate change.

Countries are assumed to be acting so as to maximize a over time collective utility function, u_i, specific for each of them, whose arguments are the domestic aggregate consumptions in each period, $C_{it} > 0$. Felicity at time t, u_{it}, is assumed to be additively separable over the periods T. The consumptions are made feasible from the production of commodities, in aggregate amounts $Y_{it} > 0$, after deducting investment decisions I_{it} and possibly diminished at each t by the domestic damages $D_{it}(\Delta_t) > 0$ incurred from the temperature change Δ_t. Thus, for each country i, the assumed objective is to maximize

$$u_i = \sum_{t \in T} \beta^{t-1} u_{it}(C_{it}) \tag{1}$$

with $0 < \beta^{t-1} < 1$ discount factors and for which at each t

$$C_{it} = Y_{it} - I_{it} - D_{it}(\Delta_t)$$

holds. In each country, production at each t results from the use of inputs such as labor L_{it} and capital K_{it}, according to some production function

$$Y_{it} = f_{it}(L_{it}, K_{it}),$$

with labor treated as exogenous (*i.e.* given) and capital resulting from accumulated investments I_{it} according to the relation $K_{it} = K_{it-1} + I_{it}$. Production also generates polluting emissions, E_{it}, the sum of which over all countries determines, in turn, the level of the worldwide externality Δ_t. One thus has for each i and t, in addition to the two equations above,

$$E_{it} = g_{it}(Y_{it})$$

and

$$\Delta_t = h(\Delta_{t-1}, \sum_{i \in N} E_{it}). \tag{2}$$

Additionally, the optimization problem is complemented by the ability of each region to reduce emissions via an (implicit or explicit) abatement cost function.

These are the bare bones of many IAMs[1]. Concerning the five IAMs in question, some are more detailed on production (WITCH) or include other activities such as trade (MICA), others being even less detailed (STACO).

As it is stated thus far, an IAM may be seen as essentially descriptive. Assuming that in all countries utility maximization is spontaneous, because occurring through competitive market forces, the models' solution is a worldwide Pareto efficient trajectory of consumptions, productions and capital accumulations when the externality of equation (2) is not present. This cannot be asserted concerning the environmental variables: due to the externality they generate, pollutant emissions and the resulting levels of ambient pollutants prevent a market world economy from achieving efficiency. If the purpose is to correct for that, the models can be taken as prescriptive and their solution interpreted as policies, that is, decision variables. Beyond Pareto efficiency other objectives may be sought for, such as coalitional stability of the solution, as done here.

2.2 The associated games

This is where game theoretic concepts can come into play, if a precise connection is made between the components of the economy and the elements that constitute a game. To that effect, remembering that a game (in strategic form) is mathematically defined, in general, as a triplet comprising a set of players, the set of strategies of the players, and the players' payoffs, let us define the international environmental externality game as

$$\Gamma = (N, (\mathbf{X}_i)_{i \in N}, (u_i)_{i \in N}) \tag{3}$$

where N, the set of players, is the set of countries, $(\mathbf{X}_i)_{i \in N}$ is a family of sets of strategies accessible to each player, and $(u_i)_{i \in N}$ is the vector of the players' payoffs, taken to be the values of the countries' collective utility functions (1).

What the players' sets of strategies exactly are may remain unspecified at this point (they do differ according to the IAM under consideration). Be it sufficient to point out now that the externality feature - a crucial one in this case - is not lost in the passage from the economy to the game: the variables E_{it} and Δ_t will

[1]To our knowledge, first formulated by Nordhaus (1994) in a world model called DICE that was not differentiating utilities across countries. This last feature – without which the issues raised in this paper do not exist – was introduced in Nordhaus and Yang (1996).

remain present in the accessible sets $(\mathbf{X}_i)_{i\in N}$ as well as the transition equation (2) that links them. For the time being and until further notice, let us denote simply by x_i any element of the set $(\mathbf{X}_i)$, and call a *strategy profile* any list, denoted x, of strategies of *all* players.

Pursuing in the direction of cooperative games, the stability literature considers players forming coalitions, that is, subsets S of N, and coordinating thereby their behavior on the externality (typically, in our economic application, participants in an environmental agreement among the coalition members). We specify this in the game by assuming that if a coalition S forms, its members have access, acting jointly, to a set $\mathbf{X}^S = \times_{i\in S}\mathbf{X}_i$ of *joint strategies* $(x_i)_{i\in S}$, on which they maximize their joint payoffs according to a social welfare function specific to the IAM. We shall denote by x^S the elements of $\mathbf{X}^S$. We simultaneously assume that the non-members maximize their individual payoffs over their respective strategy sets. Based on this twofold assumption, the outcome of the formation of a coalition is a particular *strategy profile* $\tilde{x}^S =_{def} \left((\tilde{x}_i)_{i\in S}, (\tilde{x}_j)_{j\in N\setminus S}\right)$, that Chander and Tulkens (1995, 1997) have called "partial agreement Nash equilibrium with respect to coalition S " (PANE wrt S), for which we extend the concept to allow for general social welfare functions to be maximized. This efficient solution wrt a coalition S will play a key role in the rest of this paper.

For the sake of completeness, let us point out that the formation of a coalition in the game Γ is approached differently depending upon the literature where the stability concepts that we wish to compare originate. In the PIS literature, to be a member of a coalition is treated as a strategic variable in the first stage of a two stage game, whose solution consists of, for each i, the fact of belonging or not to some coalition S (knowing that it will implement a PANE wrt that S). In the second stage the solution consists of the emission strategies of the PANE wrt the coalition chosen at the first stage. In the CS literature, Γ is treated instead as a single stage coalitional cooperative game whose core solution directly specifies which coalition forms. This difference in origins is without impact, though, on the issues and results dealt within this paper.

2.3 TU vs NTU games and transfers

The TU vs NTU distinction intervenes when a precise specification is needed for the expression of "maximizing joint payoffs" used above for describing the behavior of the members of a coalition. For this purpose, a function may be specified, called the “coalition function“[2], whose arguments are all possible coalitions $S \subseteq N$ and the image some expression of the best outcome that the members of S can achieve *as a group*, that is, using the joint strategies $\mathbf{X}^S$ they have access to.[3] And to the definition (3) of the game in *strategic* form there may be added, as a complement, a definition of the game *in coalitional form*[4] or, for short, of a *coalitional game*. We present here three alternative forms of that function which are used in the IAM literature and are the source of the distinction between TU *vs* NTU games.

If the coalition function associates with each subset S of N a real number denoted $v(S)$, this scalar being the one that maximizes *the (unweighted) sum* of the individual payoffs achieved by the members, the game is called TU and denoted (N, v) in this coalitional form. The number $v(S)$ is called the “worth“ of the coalition S. In behavioral terms – that is, in terms of strategies –, this “best“ outcome is obtained from maximizing $v(S)$ over the joint strategy set $\mathbf{X}^S$ defined above.[5] Notice that by doing so it is *also assumed* that once achieved, this maximum joint payoff is possibly redistributed among the coalition members in any way they please, by means of transfers between players inside the coalition.[6] The transfers of payoffs that we state here are usually called transfers of utility if the coalition function $v(S)$ is defined as above. Hence the acronym TU game. Among the five IAMs reviewed in the comparison paper, CWS and STACO make use of coalition functions of the kind just described and are thus TU games.

In a second form, the coalition function associates with each S a *set of vectors* of the individual payoffs, denoted $V(S)$, achieved by the coalition members playing their joint strategies. The game then is called NTU, for non-transferable utility, and denoted (N, V) in its coalitional form. The best outcome in this case

[2]We are adopting the terminology of Peleg and Sudhoelter (2007), p.9.

[3]Thus, in addition to the individual players i, coalitions are also considered as elements of the definition of the game.

[4]For a long time, actually since von Neumann and Morgenstern (1944, ChapterXI), the coalitional function was called the “characteristic function’, and the game so defined called ”game in characteristic function form“. By now, and since Aumann (1989, pp. 9 and 49), the term *coalitional* that we are using here seems to have been definitively adopted, as in Peleg and Sudhoelter (2007).

[5]Any joint strategies that yield $v(N)$ are called “efficient” strategies, and correspond to a Pareto optimum of the world economy.

[6]This is indeed desirable if there are members for which the joint strategy happens not to be beneficial in terms of their own payoff function.

is not represented by the sum of the vector's components, as it would be with with TU, but instead by the vectors itself. The reason for that lies in the underlying economic model, where the assumptions needed to justify the summation of utilities (and consequently of payoffs in the game) are considered not to be verified. In fact, the admissibility of summation or non-summation of payoffs corresponds to alternative assumptions made on the nature of the utility functions in the economy. To the TU formulation there correspond two features[7]: (*i*) that individual utilities are expressed in units of a common *numeraire* good, *e.g.* in dollar units of GDP, and (*ii*) that the utility functions are quasi-linear (*i.e.* linear in the *numeraire* commodity). Accepting these two restrictions and thus adding up unweighted utilities across individual players does not make the analysis meaningless, but it makes it dependent to quite particular interpersonal considerations.

If the two restrictions above are not accepted, identifying a best outcome for a coalition's choice of a joint strategy requires identifying, in the payoff space, vectors that do not dominate one another in the sense of vector maximization (as in Kuhn and Tucker's theorem, that is, in a Pareto sense). Such vector maximization does not yield a unique solution, in general, so that the image V of the coalition function $V(S)$ is to be taken as the *set* of undominated payoff vectors in the payoffs space[8]. In NTU coalitional games the worth of coalition S is thus represented by a set of vectors of payoffs.

The coalition function for NTU may also be defined as a single vector of the individual payoffs achieved by the coalition members playing one of their joint strategies and identified via a specific social welfare function to be maximized. The functional form of social welfare is taken to be the *weighted* sum of the players payoffs. The weights - denote them by $\lambda_i > 0$ for each $i \in S$ - are determined by considerations derived from the underlying economic model, in the spirit of some utilitarian social welfare function for instance, based on equity considerations, or in relation with the marginal utility of income of the economic agents, that the players represent. The coalitional form of the game in this case should be written $v^{\lambda}(S)$. Among the five IAMs reviewed in the comparison paper, MICA, RICE, and WITCH make use of coalition functions of

[7]As mentioned in Peleg and Sudhoelter (2007), pp. 1-2, referring to Aumann (1960).

[8]In the space of strategies, each such vector being induced by a (possibly) different joint stategy of the members of S.

the kind just described.[9] No transfers of the payoffs $v^{\lambda}(S)$ within the coalition can be envisaged, though, because of their non-comparability.

But the meaninglessness of transferring payoffs should not preclude the possibility of transferring instead some physical commodity (that induces payoffs), in the hope of obtaining, in the NTU form of the environmental externality game, stability properties similar to those well established for its TU form, which are repeated in Section 3 and applicable to CWS and STACO. We deal analytically with such "non-utility transfers" in Section 4 below, applicable to MICA, RICE, and WITCH.

Transfers play an important role in both forms of coalition functions: they shift payoffs, directly or indirectly, between the players. The reason for doing that lies, as suggested in footnote 6, in a possible discrepancy between the payoff a player obtains when implementing his part of the joint strategy x^S and the payoff attributed to him at, for example, the non-cooperative Nash-equilibrium of the game in (3). However, are the transfers themselves elements of the strategy sets?

The answer is "no" for the games associated with the five IAMs here under review. In all cases, the solution concept sought for[10] is obtained from computing first the worths of the coalitions of interest from strategy sets $\mathbf{X}^S$ that ignore transfers, after which, *ex post,* transfers between players are computed so as to find (if possible) a vector of payoffs of all players that achieves the said solution concept. With this methodology, the virtue of the transfers, on which the computational experiment of the Lessmann et al. (2014) paper reports, is conceptual rather than behavioral. They serve to appreciate the feasibility of the stability concepts, but are not an expression of behavioral assumptions on the players.

We now move, in sections 3 and 4, to the detailed analysis of coalitional stability, respectively for the TU version and the NTU version of the environmental externality game.

[9]For analyses of these models with λ_i-weighted utilities but without the application of transfers see Lessmann et al. (2009) for MICA and Yang (2008) for RICE. Bosetti et al. (2013) analyze stability results in WITCH and consider transfers by redistributing the present value of consumption rather than utility (see discussion below equation 11).

[10]Namely, a potentially internally stable strategy for some coalitions, or a core stable strategy for the grand coalition, which will be described in detail in sections 3 and 4 below.

3 The Stability Concepts in the TU-case

3.1 Potential Internal Stability (PIS)

The PIS concept derives from the notion of internal stability (IS) introduced in the international environmental game by Carraro and Siniscalco (1993) and Barrett (1994). As mentioned in Section 2.2, a two-stage cartel formation game is defined where in the first stage players decide on being part of a coalition. In the second stage the players implement the[11] PANE wrt the so chosen coalition. This defines the equilibrium vector of strategies in Γ.

IS is then a property of the coalition chosen at the first stage. It is verified if for each of its members the payoff of being inside the coalition is higher than when being outside of it, and thus higher than when being a free-rider *vis-a-vis* the remainder of the coalition. Formally, and more precisely:

$$\text{A coalition } S \text{ is internally stable if } \forall i \in S,\ u_i^S \geq u_i^{S\setminus\{i\}}, \tag{4}$$

with u_i^S the payoff of player i at the PANE wrt to the coalition S when i is a member of it, and $u_i^{S\setminus\{i\}}$ player i's payoff as a singleton at the (new[12]) PANE wrt the coalition $S \setminus \{i\}$ that i is no longer a member of.

Three points need to be made. First, note that in each of these two situations, the payoffs u_i^S and $u_i^{S\setminus\{i\}}$ result from joint strategies chosen by the coalitions S and $S \setminus \{i\}$ respectively. For a given S, verifying the right hand side of (4) requires re-computing as many other PANEs as there are members in S. Thus, when the number of countries is large, and the number of coalitions is, in turn, very large, the computational task of checking for IS for each coalition ends up being quite considerable. Next, notice that, as stated, this definition of IS is independent of whether or not the payoff functions are quasilinear and TU or the coalition function is represented by v^λ and payoffs are NTU, as discussed in section 2.3 above. Finally, the payoffs mentioned in (4), as well as the strategies

[11]For every coalition S, this equilibrium is proved to be unique by Chander and Tulkens (1997) in the usual analytical form of the environmental externality game, which is independent of IAMs. When used in the present applications to IAMs, the uniqueness of such equilibrium is assumed, and much needed as seen in the sequel.

[12]Because the defection of i induces the remaining members of S to adapt their equilibrium strategies.

that induce them, bear on the whole period $1\ldots$T: thus, by construction of the model, coalitions remain fixed over time.

PIS is a weakening of the IS requirement. It is defined as follows (see Carraro et al. 2006):

$$\text{A coalition } S \text{ is potentially internally stable if } \sum_{i\in S} u_i^S \geq \sum_{i\in S} u_i^{S\setminus\{i\}}. \tag{5}$$

If this inequality is satisfied, the total payoffs of the coalitions' members, *i.e.* the left hand side, is sufficient to cover the total of the free-rider payoffs, *i.e.* the right hand side, that all these members might claim simultaneously, even if for some members condition (4) does not hold. Therefore, as observed by Carraro et al. (2006), if (5) is satisfied, a scheme of payoff transfers among the members of S – from those for which (4) holds with strict inequality to those for which (4) does not hold – can be devised such that, by implementing such a scheme, the coalition S would become internally stable in the sense of (4). Hence the *"potential"* qualification. The transfers are not explicitly stated by these authors, but Weikard (2009) has formulated conditions that they should satisfy for a specific analytical model.

There is in particular a feasibility condition: for PIS to prevail, the total of transfers, formulated in units of payoffs as done above, should not exceed the (positive) difference between the two sides of the inequality (5). If that difference is negative, the sum of the members' payoffs on the left hand side is not sufficient to cover the claims stated by these same members in the right hand side. Then, PIS does not hold among the members of coalition S.

For a given game, whether or not the PIS property is satisfied by all coalitions, by only a few, or perhaps by no coalition at all, depends on the specifics of each game (that is, properties of their strategy sets and payoff functions), and within each game, on the specifics of the members of each coalition.[13] It may hold for some coalitions, and not for other ones. Weikard (2009) shows that within a simple analytical model large heterogeneity in damages from emissions among the members induces a coalition to become PIS. To the best of our knowledge, no general analytical condition has been formulated so far that guarantees the existence of PIS coalitions in more complex versions of the environmental ex-

[13]In view of the remark that prompted the next to last footnote, notice that the summand on the right of (5) bears on a number equal to $|S|$ of possibly different PANE's.

ternality game.[14] However, in numerical simulations of the IAM type, Carraro et al. (2006), as well as Brechet et al. (2011) found some of them, and these findings led to the Lessmann et al. (2014) comparison paper, seeking for determinants of potential internal stability of coalitions.

Because the PIS concept is formulated in terms of sums of payoffs, and such sums are economically meaningful only if payoffs are quasi-linear, as argued in Section 2.3, the concept has been used in IAMs that assume TU. To overcome this limitation, we seek in Section 4 below for an extension of PIS to the NTU case.

3.2 Core Stability (CS)

When Γ is treated as a coalitional game, the fact that players form coalitions is not rendered explicit as a distinct strategic variable as in the case of IS. Instead, it is included as part of the solution concept of the game.

The core of a coalitional TU game (N, v) is defined as the set of payoffs vectors $u^{**} = (u_1^{**}, \ldots, u_n^{**})$, feasible for coalition N, such that (see Myerson (1991) p.462)

$$\forall S \subseteq N, \sum_{i \in S} u_i^{**} \geq v(S). \tag{6}$$

By feasibility is meant that N, the grand coalition, has access to some joint strategies[15] $x^{**} = (x_1^{**}, \ldots, x_n^{**})$ for its members[16], which yield $u_i(x^{**}) =_{def} (u_i^{**})$ for each of them, and satisfies

$$\sum_{i \in N} u_i(x^{**}) \leq v(N). \tag{7}$$

When equality holds in (7) the core joint strategy x^{**} is an efficient one, and (6) says that for the members of any coalition S, this strategy profile yields aggregate payoffs (the LHS of equation 6) larger than the best this coalition can possibly do (the RHS of equation (6)). Thus, no coalition can pretend to "block" the proposed vector u^{**} or to improve upon it. The vector u^{**}, as well

[14]Brechet et al. (2011, pp. 62-63) provide logical arguments explaining why PIS may not hold in general.

[15]There may be many.

[16]since $S = N$ in this case, $x^{**}$is also a strategy profile.

as the coalition N and the strategy profile x^{**} that induces it, are stable in that sense.

Defined in this way, the core appears as a set in the space of payoffs and in the strategy space as well. From the use of the additive coalition function in the specification of the non-blocking condition (6), the possibility of payoff transfers inside coalition N is implicitly at play. Indeed it was stated earlier that "the joint payoff is possibly redistributed among the coalition members in any way they please". However, such transfers may not be necessary. This is the case if for all the members of N the efficient strategies x_i^{**} happen to yield individual payoffs that directly meet (6). The simplest example of that is when all players are assumed to be identical: with the standard analytical model, Chander and Tulkens (1995) show, in their Section 6, that an efficient strategy profile without transfers is in the core in that case. We leave it as a conjecture whether in anyone of the five IAMs, with their parameters modified so as to be "symmetric" (that is, countries being identical), one could obtain the same result.

Thus, CS does not rest in an essential way on the notion of transfers, as it is sometimes believed. However, when the CS property is not met by an efficient strategy in an environmental game, the claim of Chander and Tulkens (1995, 1997) is that the particular transfers they define are a sufficient instrument to obtain stability. While this was first established for the analytic model in its static version, an extension to a formulation adapted to the intertemporal framework of IAMs led to the formula of "generalized GTT transfers" given in equations (30) and (31) of Eyckmans and Tulkens (2003). Introducing it in the simulations done with CWS, and repeating the exercise with Brechet et al. (2011) has revealed that the optima so computed, supplemented by the generalized GTT transfers, are indeed solutions in the core of the CWS game. To our knowledge, there has been so far no reporting of tests of that property with the other TU-IAM, namely STACO.

As a final remark, since the games constructed from CWS and STACO are TU games, the transfers are expressed, as suggested above, in units of payoffs, that is, in units of discounted sums of aggregate consumptions in the various countries over the whole time horizon of the models, see equation (1). This raises the practical question of *when* these sums are to be paid and received. At whatever times they are paid, the amount of consumption at these points in time needs to be sufficient to ensure the feasibility of the transfer (formalized below in equation 13). The same question of when transfers are paid arises in the next section and we postpose the discussion of it to the concluding section of the paper.

The extension of the PIS and core stability concepts to the NTU case will be our second purpose in the next section.

4 The Stability Concepts in the NTU-case

In this section we present how the two stability concepts translate to the NTU setting with a transferable commodity. We take advantage of the fact that in the underlying economic model there is at least one commodity which is transferable and yields utility, and we use this one for transfers. As was the case for TU transfers in the preceding section, commodity transfers are treated below "ex post", that is, they are not part of the strategy sets of the players, but are introduced after an optimum without transfers is obtained, as a computational operation with no explicit behavioral motivation on the part of the players.

4.1 Differences between transfers in the TU and the NTU games

Transfers in our model are just numbers denoted $\tau_{it} \in \mathbb{R}$, satisfying the condition that one unit of it added to a player i has to be balanced by an equal reduction from another player, *i.e.* and more generally, for any transfer we have $\sum_i \tau_i = \mathbf{0}$, with $\tau_i = (\tau_{it})$. As we wish to restrict transfers to occur within coalitions, the set of all possible transfers we shall admit as possibly occurring when S forms reads:

$$\begin{aligned} \mathcal{T}(S) = \{(\tau_i) = (\tau_{it}) : i \in N,\, t \in T \quad & \wedge \quad \forall t \in T : \sum_{i \in S} \tau_{it} = 0 \\ & \wedge \quad \forall j \notin S : (\tau_{jt}) = 0\}. \end{aligned} \tag{8}$$

Both the units in which transfers are expressed as well as the way in which transfers enter the players' payoff functions differ depending upon the TU vs NTU formulation of the game. In the former, we repeatedly mentioned above that transfers are implicit in the definition of coalition functions as well as in the results obtained with them. To make them explicit we may rewrite the individual payoff function (1) of player i when he is a member of S and after transfers as:

$$u_i^{TU,S} = (\sum_{t \in T} \beta^{t-1} u_{it}(C_{it})) + \tau_i^{Su}, \tag{9}$$

in which indeed the transfer τ_i^{Su} (> 0 if received by i, < 0 if paid by i) is expressed in the same units as the discounted payoff, and done *ex post*, that is, as a lump sum separate from the choice of C_{it}, hence the superscript u and the absence of a time index.[17]

In the NTU formulation, one may propose to write[18] the players' individual payoff functions as:

$$u_i^{NTU,S} = \sum_{t \in T} \beta^{t-1} u_{it}(C_{it} + \tau_{it}^{Sc}) \tag{10}$$

where the transfer is now denoted τ_{it}^{Sc}, expressed at each time t in the same units as the composite commodity (hence the superscript c). Discounted over the whole time period T with discount-factor r_{it}, it amounts to the single number that we now define as

$$\bar{\tau}_i^{Sc} = \sum_{t \in T} r_{it} \tau_{it}^{Sc}, \tag{11}$$

as done in Bosetti et al. (2013).[19] Bosetti et al. add the discounted transfers in equation (11) to the discounted sum of consumption streams $\sum_{t \in T} r_{it} C_{it}$, an approximation to utility, in order to test the PIS-property of a coalition. This procedure however neglects the fact that marginal utility is non-constant and can therefore not provide a definite test of whether a coalition is PIS via ex-post re-distribution. The definition of PIS given below is based on utility rather than discounted comsumption and is therefore more general.

The role that the transfers defined in equation (10) play in the two stability concepts is presented in the next subsections.

[17]This is exactly what Eyckmans and Tulkens (2003) do for CWS (see their equations (30) and (31), which are more explicit on the time dimension than equation (1) in Brechet et al. 2011), and repeated in the Lessmann et al. (2014) comparison paper. The same applies to STACO as appears from formulae of their transfers given as equations (13) and ff. in the Nagashima et al. (2009) version of the model, and repeated in the comparison paper.

[18]As done for WITCH in Bosetti et al. (2013), see their equation (4), and also in the Lessmann et al. (2014) comparison paper.

[19]See in particular equations (7) and (8).

4.2 Potential Internal Stability in IAMs under NTU

For any coalition S, IS under NTU is defined exactly as in equation (4) when the game is of v^{λ}-type. For PIS, since we cannot use the summation of utilities as in equation (5), we express the definition of the concept in the NTU case in terms of vectors of payoffs as determined by the functions (10).

As these result from strategy profiles specific to each coalition S, we have to rephrase (5) in a way that expresses them fully. This is done as follows: A coalition S is potentially internally stable in the v^{λ} environmental externality game if, given the PANE $\tilde{x}^S$ that prevails when coalition S forms as well as, for each $i \in S$, the PANE $\tilde{x}^{S\setminus\{i\}}$ prevailing in the coalition $S \setminus \{i\}$ that forms when player i defects from S, there exists a transfer scheme $(\tau_i^{Sc}) \in \mathcal{T}(S)$ and inducing payoffs $\tilde{u}_i^{NTU,S}$ such that

$$\begin{aligned}\forall i \in S,\ \tilde{u}_i^{NTU,S} &=_{def} \sum_{t\in T} \beta^{t-1} u_i(\tilde{C}_{it}^S + \tau_{it}^{Sc}) \\ &\geq \tilde{u}_i^{NTU,S\setminus\{i\}} = u_i(\tilde{x}_i^{S\setminus\{i\}}),\end{aligned} \tag{12}$$

where $\tilde{C}_{it}^S$ is the it-th consumption component of the strategy profile $\tilde{x}_i^S$.

As seen in this last expression the commodity transfers (8) can replace the utility transfers for coalition S to ensure its members at least their outside option payoffs $\tilde{u}_i^{S\setminus\{i\}}$. Doing so a feasibility issue arises in commodity terms: the amount of commodity to be transferred by the members of S to the possibly defecting members at each point in time must be feasible for this task, the amount being taken from the commodities that coalition S produces. Since transfers are redistributed in an ex-post fashion without influencing the underlying economies at the PANE wrt S, the amount of commodity paid by any region has to be deducted purely from consumption so that investment decisions can remain at their respective level:

$$\tilde{C}_{it} + \tilde{\tau}_{it} > 0 \quad \forall i \in S \quad \forall\, t \in T. \tag{13}$$

This feasibility constraint applies to all IAMs. Additionally and depending on the specific modeling context, other constraints may be added to ensure that ex-post redistribution is feasible for all economies.

In Bosetti et al. (2013) results as computed with WITCH are reported on PIS holding for (some) coalitions, albeit basing the redistribution on the present value of consumption and transfers defined as in equation (11). Typically, the grand coalition is never PIS (as indicated on the second and last lines of their Table 3), but various smaller ones are. Other IAMs with NTU have thus far not been subject to PIS analysis with coalitions.

As a closing remark on PIS, applying this NTU-definition to a TU-game should lead to the same conclusions as when using the simple TU definition in (5), due to the fact that in both cases transfers are ex-post and do not influence the solutions $\tilde{x}^S$: only a fixed 'cake' is distributed. Yet the cake is of a different composition as well as subject to different limitations as to its size. Therefore it is not clear whether a same game, treated successively as TU and NTU would lead to identical coalitions being either PIS or not.

4.3 Core stability in IAMs under NTU

The core of a coalitional NTU game (N,V) is defined as the set of payoffs vectors $u^{**} = (u_1^{**}, \dots, u_n^{**})$, feasible for coalition N, which are not dominated by feasible payoff vectors of the members of any other coalition. Formally,

$$\forall S \subseteq N, \text{ and } \forall i \in S, \; u_i^{**} \geq u_i^{*S} \tag{14}$$

needs to hold for all feasible payoff vectors $(u_i^{*S}) \in V(S)$ of the coalitions S.

More explicitly, in terms of strategies, by "no domination" is meant that N, the grand coalition, has access to some joint strategies[20] $x^{**} = (x_1^{**}, \dots, x_n^{**})$ for its members[21], which yield $u_i(x^{**}) =_{def} (u_i^{**})$ for each of them, and is such that there exists no coalition $S \subset N$ for which a PANE wrt S, $\tilde{x}^S$, yields

$$u_i(x^S) \geq u_i(x^{**}) \; \forall i \in S \text{ with} > \text{for at least one } i. \tag{15}$$

For $S = N$, expression (15) replaces, in this NTU definition of the core, the efficiency requirement (7) in the TU case. For all other S, equation (15) ensures that no coalition can "block" the proposed vector u^{**} by pretending to improve

[20] There may be many.

[21] since $S = N$ in this case, x^{**} is also a strategy profile.

upon it. The vector u^{**}, as well as the coalition N and the strategy profile x^{**} that induce it, are "NTU-stable" in that sense.

Turning to IAMs, the issue of interest is to find out whether a computed efficient solution could belong to the NTU core, with or without transfers (and not, as is sometimes believed, to compute the entire core). To that effect, let us briefly recall how this is done in the TU framework, *e.g.* with CWS[22]. Compute first the efficient solution (with all weights equal to 1) , and record in particular the payoff of each player at that solution. Compute next the worth of each coalition (with the same equal weights), that is, a single number for each coalition, and compare this number with the sum of the payoffs that the coalition members obtain at the efficient solution, another single number. If the difference between the former and the latter is positive for all coalitions, the efficient solution already belongs to the TU core, without transfers. If this difference is negative for one or more coalitions, it does not, but after applying GTT transfers to the efficient solution, and making the comparison again, it appears that the difference is positive for all coalitions. Thus, efficiency with transfers yields a solution in the core of the game associated with CWS, for the parameter values used in that experiment.

In the NTU framework, parts of that procedure can be followed, while others cannot for the obvious reason of non-additivity of payoffs. Nevertheless, let us follow that path. Take an IAM, compute a PANE of the Grand Coalition, and record the individual payoffs so obtained, as well as, of course, the strategy profile that induced them. Check whether or not these strategies, after having been introduced as arguments of the individual (NTU) payoff functions, are undominated in the sense of (14). If they are undominated, we claim that the solution is in the NTU core.

For the case of ex-post transfers $(\tau_{it}) \in \mathcal{T}(S)$ that induce feasible payoff-vectors of the sub-coalitions, a procedure to test for the blocking power of a coalition S is proprosed in section 5.3. With this method, a payoff vector u^N of the Grand Coalition can be rejected to belong to the core if a blocking coalition is found with respect to the case that a transfer scheme $(\tau_{it}) \in \mathcal{T}(S)$ exists that, once implemented, violates equation (14). However, since not the entire accessible strategy space is tested in this manner, only potential candidates of u^{**} can be identified.

[22]see the details in Table 5, pp. 321-322 of Eyckmans and Tulkens (2003).

To our knowledge, none of the five IAMs here under review has been subject to CS analysis with NTU coalitions[23].

5 Testing for stability

5.1 A method to test for PIS

In case of NTU of the v^λ-type (models MICA, RICE, and WITCH), we can test for PIS of a coalition by considering all possible transfer schemes as control variables in a maximization problem. Indeed, at the PANE wrt to some coalition S, given the corresponding utility vector $(u_i^S)_{i\in S}$, take as fixed the vector of consumptions $\tilde{C}^S$ of its members. Then, for an arbitrary but fixed coalition member k solve

$$\max_{(\tau_{kt})} \quad u_k(\tilde{C}^S_{kt} + \tau_{kt}), \quad (\tau_{it}) \in \mathcal{T}(S) \tag{16}$$

$$\text{subject to} \quad u_j(\tilde{C}^S_{jt} + \tau_{jt}) \geq u_j^{S\setminus\{j\}} \quad \text{for } j \in S \setminus \{k\} \tag{17}$$

$$\tilde{C}_{it} + \tau_{it} > 0 \quad \text{for } i \in S, \quad t \in T \tag{18}$$

$$\ldots.$$

That is, redistribute consumption of all $i \in S$ at each time t such that all members except player k receive at least their outside payoff, and player k receives as much as possible given this restriction. Additional feasibility constraints can be added to this procedure that are specific to the IAM. Then S is PIS if the payoff $\tilde{u}_k^S = u_k^S(C^S_{kt} + \tau^*_{kt})$ of k, with (τ^*_{kt}) the solution to (16–17), exceeds her outside payoff $u_k^{S\setminus\{k\}}$.[24]

5.2 A continuous measure of PIS

The excess surplus per member of the coalition, by which we mean any surplus that is not necessary to make the coalition PIS, may serve as a continuous indicator of the degree of stability.

[23] In the Lessmann et al. (2014) comparison paper, CS is not dealt with at all, but is announced for later.

[24] This approach is similar to the linear programming problem proposed for example in Friedman (1986, pp. 187-217) to test whether the core is nonempty in TU games.

For $j \in S \setminus \{k\}$, the $(\tilde{C}^S_{jt} + \tau^*_{jt})$ are *minimal* in the sense that they generate no more than the necessary outside payoff. By maximizing the left-over consumption

$$\max_{\Delta C_t} \quad \Lambda = \sum_t \frac{1}{1+r_{kt}} \cdot \Delta C_t \tag{19}$$

$$\text{subject to} \quad u_k(\tilde{C}^S_{kt} + \tau^*_{kt} - \Delta C_t) \geq u_k^{S\setminus\{k\}} \tag{20}$$

$$\Delta C_t \geq 0 \tag{21}$$

$$\tilde{C}^S_{kt} + \tau^*_{kt} - \Delta C_t > 0 \quad \text{for } t \in T \tag{22}$$

$$\ldots.$$

while ensuring the arbitrary but fixed member k from (16–17) at least her outside payoff, the maximum amount of money still to be distributed among the members is determined, where r_{kt} is the discount-rate of member k. If S is not PIS, one may drop the conditions (21) and allow Λ to be negative. This then indicates how much consumption S is lacking. The per-member indicator $\Lambda/|S|$ resembles the continuous measure.

5.3 An algorithm to test for Blocking Power

Consider a feasible payoff vector $(\tilde{u}^N_i)_{i\in N}$, possibly induced by a transfer scheme $(\tau_i) \in \mathcal{T}(N)$ in the grand coalition N, and a potentially blocking coalition S. The question then is whether there exists a transfer scheme $(\tau_i) \in \mathcal{T}(S)$ that ensures every member $j \in S$ at least her inside-payoff $\tilde{u}^N_j$ so that (14) is not fulfilled. In order to derive this, we set up a maximization procedure. Choose an arbitrary but fixed member $j \in S$. Maximize her payoff with arbitrary transfers $(\tau_i) \in \mathcal{T}(S)$ subject to the constraints that every other member $k \in S \setminus \{j\}$ receives at least the payoff $\tilde{u}^N_k$:

$$\max_{(\tau_i)} \quad u_j(\tilde{C}^S_{jt} + \tau_{jt}), \quad (\tau_i) \in \mathcal{T}(S) \tag{23}$$

$$\text{subject to} \quad u_k(\tilde{C}^S_{kt} + \tau_{kt}) \geq \tilde{u}^N_k \quad \forall k \in S \setminus \{j\} \tag{24}$$

$$\tilde{C}^S_{kt} + \tau_{kt} > 0 \quad \text{for } t \in T \tag{25}$$

$$\ldots.$$

with $\tilde{C}_{jt}^S$ being the jt-th component of a strategy profile $\tilde{x}_j^S$ that the coalition S has access to. This ensures that every member $k \in S \setminus \{j\}$ receives the inside payoff $\tilde{u}_k^N$ and that member j will get the highest payoff possible under these constraints. If the resulting payoff of player j is greater than her payoff in the grand coalition, $\tilde{u}_j^N$, then the solution to problem (23), $(\tilde{u}_i^S)_{i\in N}$, contradicts (14) and the coalition S can block the grand coalition.

6 Concluding remark

A "practically" minded reader might ask the question: when are these stability enhancing transfers ever paid and received? The dynamic nature of the ex-post transfers we propose in the NTU formulation of IAMs reveals that they need to be paid over the entire time horizon of the model. Therefore, just as the agreement of a stable coalition prescribes abatement efforts for every period, transfers are specified as binding for every period as well. For the case of TU, transfers are formulated in discounted present value terms, which makes it possible that they are paid in the final period after all benefits and costs have accrued. However, also in this case a dynamic formulation based on the consumption streams available to the regions could be feasible. Now, given the extremely long extension of that time path, the decisional aspects of the optimizations achieved by IAMs such as those reviewed here cannot be considered as realistic.

There is nevertheless an interest in defining and computing them as they confirm and illustrate the qualitative assertions of the theory, and give some orders of magnitude of their quantitative importance, relative to the rest of the economy.

Yet, the lack of realism just mentioned is not to be found in essential properties neither of the stability concepts nor of the transfers instrument. It is rather due to the methodology of open loop dynamic optimization used in all five IAMs. Other techniques have been proposed to avoid the rigidities of this method, but their implementation in numerical IAMs is currently at a exploratory stage only.

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